Oral exam, applied and computational mathematics, all-around, 2013

Let $D_n = \{x \in R^n : |x| < 1\}$, f be a smooth map from $\overline{D_n}$ to $\bar{D}_n$. For homotopy $H : \overline{D_n} \times [0,1] \to R^n$

$$H(x,t) = (1-t)(x-a) + t(x - f(x))$$

we can find $x = f(x)$ from $H(x,t) = 0$. Here $a \in D_n$.

1. Prove $H^{-1}(0)$ is a smooth curve for almost all $a \in D_n$. (Hint: Sard's theorem and implicit function theorem)

2. Prove this curve starts from $x = a$ will stay inside D_n for $0 \leq t < 1$ and when $t = 1$ yields x_0 s.t. $f(x_0) = x_0$.

3. How to follow this curve? (Hint: parametrize the curve by arc length)